Geometric Modeling of A Cloth Draping System

by

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Certificate

This is to certify that the work contained in this thesis entitled **Geometric Modeling of A Cloth Draping System** by **T. Adinarayana Reddy** (**Roll No: 9311128**), has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

15/3/95

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Abstract

An attempt to develop and design a method for predicting the drape of a fabric garment on a mannequin is made. Various physical properties like bending, shearing and cloth weight are considered while the cloth is being draped. Physical properties of the cloth are mapped into geometric constraints of a three dimensional mesh which is an approximation of a cloth piece to be draped. The points of the mesh, which are relaxed so as to satisfy the geometric constraints, give the description of the draped cloth. Then the individual yarns are approximated from this coarse mesh and rendered to give a translucent effect.

The geometric model developed has been used to render different kinds of fabrics having different physical properties. The proposed approach will serve as a predictive tool in apparel design.

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Chapter 1

Introduction

1.1 Modeling of Deformable Surfaces

The problem of modeling deformable surfaces is receiving considerable attention of researchers in the area of geometric modeling. Deformable models having elastodynamic characteristics can be used in simulating many physical systems. The technique of metamorphosis, now being used extensively in computer animation, also requires a time dependent deformable model. The problem of modeling and rendering statically as well as dynamically deformed surfaces seems to be of interest to computer animators as well as design engineers.

The general approach of describing such deformable surfaces consists of using the equilibrium equations of continuum mechanics. The deformation of a surface can be predicted using a solution of such equilibrium equations. It is assumed that the undeformed geometry of a surface, the loading pattern, the support conditions and the material properties of the surface are known. The solution of equilibrium equations poses a formidable numerical computational task.

The numeric procedures seem to be either using a discretization strategy such as F.E.M. [25] or the optimization strategy such as finding the geometry of a deformed surface using minimal energy principle [26]. This approach is useful in the engineering analysis. However, it has not been found to be suitable for either design-synthesis

work or for the purpose of predicting quickly the geometry of a deformable surface.

Another approach used for modeling deformable surface uses the concept of lattice structure [15]. From computation view point, this approach is economical. In this methodology a polyhedral lattice structure surrounding the undeformed surface is defined. A subsequent change in the lattice structure then affects the surface linked with the structure. The major drawback of this approach is that the designer is not able to predict the extent to which the geometry of a surface gets deformed by the lattice structure in a specific manner.

1.2 Fabric Drape Surfaces

Draped surfaces are a type of deformable surfaces. Most of the researchers in the field of deformable surfaces tried to model and simulate cloth materials. Modeling, animating and realistically representing cloth objects have been generating much interest in the field of computer graphics in the recent times. One useful outcome from this increased level of interest in this area would be its direct contribution to enhancing the development and use of CAD/CAM systems for garment industry. Most of the current research efforts on modeling cloth material concentrate on the formulation and representation of shapes and states of the cloth.

The problem of visualizing the drape of a fabric on a given support structure is of importance to fashion designers or apparel designers [4] [6]. Most of the approaches can be roughly classified into three groups. One group focuses on the *geometric approach* which describe the shape of the object entirely as continuous functions as illustrated by Weil [26]. Other group focuses on the *physical approach* which describes the state of the cloth by considering the deformation of the cloth based on some variational principles such as Lagrange form or D'Alembert's form [25] [27] [12] [2]. Such approach result in dynamic equations which resemble the equations frequently employed to study the deformation structures in vibration and structural analysis. The third group focuses on the *combined approach* where some part of the physical modeling is considered while taking the geometry of the cloth into account [5].

An innovative approach addressed the modeling of draped fabric surfaces using intrinsic geometric parameters to define the geometry of a draped fabric surface [5]. The curvature versus arc length description of a curve reflects typically the bending moment diagram that one encounters in defining deformations of flexural members. Moreover, the draped fabric is described as a swept surface where the directrix describes the geometry of the supporting surface and the generatrix describes the drape of a fabric. This model is good for simple supporting structures and is not very much suitable for complex supporting structures like mannequins.

Current approach maps the physical properties of a cloth into geometric constraints of a mesh which is a representation the fabric to be draped. The physical properties which are of considerable importance in cloth-draping are stiffness, shearing properties and weight density of a cloth. All the above-mentioned physical properties are simulated by imposing the geometric constraints on the quadrilateral mesh. The node points of the mesh are relaxed so as to satisfy the geometric constraints and the relaxed mesh gives the description of the draped fabric.

1.3 Problem Statement

Drape is an important consideration to a fashion designer. The idea is to generate a predictive tool for the fashion designers to see the drapings of various cloths with different physical properties. The physical properties to be considered here are shearing, bending and cloth weight. The problem is to simulate the draping behaviour of a cloth piece when it is placed on a mannequin and to give an outline of fitting a cloth on a general surface of double curvature. Current approach models the cloth as a quadrilateral mesh of connected points. The physical properties pertaining to the cloth to be draped are specified by the user. Current approach maps the physical properties into geometric constraints of the cloth mesh. The physical properties considered here are bending, shearing and cloth weight. By relaxing the mesh points, an approximate description of the draped fabric is obtained. The drape so obtained is rendered to get effective rendering of the cloth. An outline is given for fitting a cloth

on a general non-uniform rational B-spline (NURB) surfaces.

1.4 Organization

In the forthcoming chapter, behaviour of cloth materials, physical properties of the cloth and their mapping into geometric constraints is given. Apart from these, an introduction to Drape and the physical properties affecting it are described.

The third chapter deals with the graphic simulation of the drape of various cloths is described. The algorithms used for mapping physical properties into geometric constraints of the cloth-mesh and relaxation of the mesh points is described.

Rendering a cloth in three dimensions to give realistic appearance is described in the fourth chapter. A rendering method which renders individual yarn of a cloth to give a semi transparent look to the cloth, is described.

Fifth chapter deals with the fitting problem where a general method is described for fitting a cloth ply on to any non-uniform rational B-spline surface (NURBS).

System design and implementation issues are covered under sixth chapter.

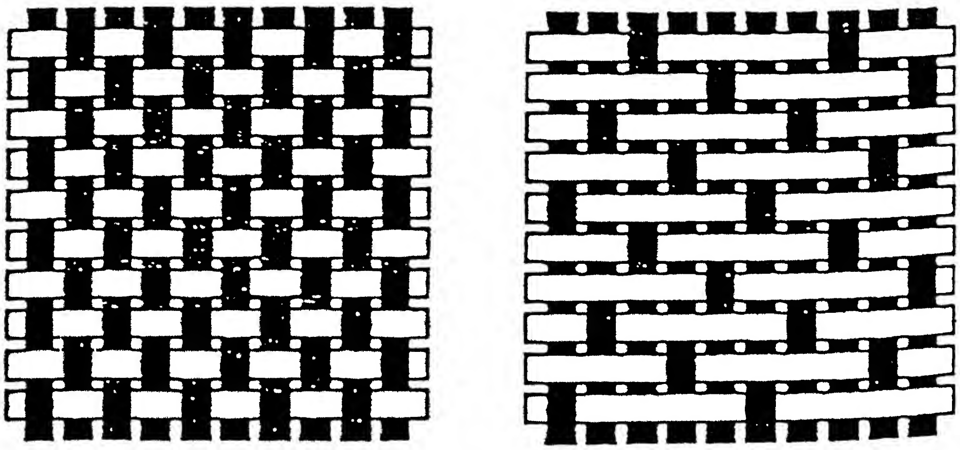
Chapter 2

Drape of a Cloth

2.1 Properties of a Cloth

Basically a cloth is woven as an interconnected matrix of perpendicular yarns. When the cloth is held vertically, a horizontal yarn is called *warp* and a vertical yarn is called *weft*. Most of the materials, when bent in one direction, resist bending in other directions. As an example, consider a rectangular paper which is bent along its breadth. It resists bending along its length. Such materials can not be bent in a doubly curvatured surface easily. But there are certain materials like cloth and rubber which can be bent in any direction and are very much suitable for bending them into doubly curved surfaces. In order to make a doubly curved surface out of such materials, an external force has to be applied on them. Rubber gets accommodated to the area changes that occur while being deformed into doubly curved surface by varying its thickness at the appropriate places, while a cloth gets sheared when it accommodates to area changes. *Shearing* is the property of the cloth where the warp and weft yarns in the cloth move such that they are not at right angles to each other. Cloth gets sheared very easily by applying even a small load on it. The load required to shear a cloth is so low that it gets sheared because of its own weight, when a cloth is hung freely.

It is this fantastic property which gives an aesthetic look to the folds of a cloth



a b

Figure 2.1: a. Plain Weave b. Satin Weave

when it is placed on a support geometry or hung freely.

Cloths are generally classified in various ways depending on the type of the weave, type of the material used etc. One form of the weaving where the warp and weft are interwoven such that a warp and a weft alternate is called *Simple Weave.* A weave in which either a weft or a warp dominate is called *Satin Weave.* (See Figure 2.1)

Various cloths differ because of their physical properties and texture. Major physical properties of the cloth are bending, shearing, drape, weight, elasticity, thickness, shrinkage, inter-yarn distance and the number of warp-weft crossings per unit area.

Bending is generally measured in terms of *bending length* and *bending stiffness coefficient.* Bending length is generally measured by a cantilever test or a heart loop test. Shearing is generally is measured in terms of *shearing coefficient.* A shearing coefficient is measured by specially mounted strain guages.

2.2 What is a Drape?

When a cloth is hung or laid on a supporting structure, it folds under its own weight to give a pleasing look to it. These folds are called *drape* of the cloth.

Drape of a cloth is its physical property. It plays an important role in cloth and garment industries as various researches show a strong relationship between the number of purchases of garments and the extent to which they drape to give a pleasing look.

Drape is measured using a *drape meter*. The principle of drape meter is the placement of circular fabric specimen between two small circular plates that are supported. The annular ring of the cloth not held between the plates is then free to drape over the edge of the support. A light and lens located below the specimen is used for projecting the shadow of the cloth specimen upwards. The image is traced on a paper and cut out. Drape is calculated in terms of a *Drape Coefficient*, which is defined as the percentage of the annular ring of fabric (less the supporting ring) obtained by vertically projecting the shadow of the drape specimen (less the supporting ring).

The extent to which a cloth drapes is inversely proportional to the drape coefficient. If the drape coefficient is low, it gets draped to a large extent and if high, it gets less draped. Generally stiff materials have high draping coefficient while loose materials have lower drape coefficient(i.e. higher drape). The stiff materials exhibit less drape as they resist bending and hence have high draping coefficient.

Drape coefficient is dependent on the bending length, bending stiffness, shearing stiffness and the weight of the cloth, the radius of the disks of the drape meter etc.

Drape of a cloth can be assessed by using the number and the geometry of the folds or nodes of the cloth which is being tested on a drape meter. Highly drapeable fabrics will form more number of folds when compared to the stiffer cloths. Another alternative way of assessing the drapeability is the size and shape of the nodes. Large nodes with a lower degree of curvature are formed in less drapeable surfaces.

2.3 Fabric Properties Affecting Drape

Fabric properties which have been shown to affect drape are bending and shearing, and also fabric weight which takes into account the gravity force on the draping specimen. In order to form folds easily, a fabric should have a low resistance to bending. A cloth bearing a high stiffness has a less tendency to drape where as a cloth with low stiffness has a higher tendency.

If the curvature is there in more than one planar directions, as it is in complex draping patterns that require the fabric to bend in two directions, then the fabric must undergo some shear deformation. Shearing occurs when the yarns in the fabric move relative to one another. In woven fabrics, this means that the right angle orientation of warp and weft yarns is altered. Drape is majorly affected by the shearing property of the cloth. A cloth with high shearing properties has high drapeability. Similarly a cloth with less shearing property has low drapeability.

Cloth weight is nothing but a constant downward pull on the cloth and hence there is a direct relationship between the drape and the weight of a cloth. A cloth with high weight has more drape when compared to the lighter cloth with the same bending and shearing coefficients.

2.4 Mannequin

A *mannequin* is a life sized model of a human body. Mannequins are generally seen in the cloth's show rooms, fashion designer offices and tailor shops. They are generally used for displaying clothes on them and for seeing the draping effects of clothes.

For our purpose of showing a cloth's draping, a mannequin model has to be generated in the computer. A mannequin can be assumed to have bilateral symmetry. This assumption, even though scientifically wrong, will simplify the design of a mannequin. We can define half section of a mannequin and can take a mirror image of it to get the other half-section.

A mannequin model can be designed, mostly, by using any of the two following forms.

- Parametric Form
- Polygonalized Form

A mannequin can be designed in a parametric form. Surface of the body can be defined using parametric surfaces like B-Splines etc. Body of the mannequin can be designed by specifying control points and knots for the B-spline surface in 3D space. By changing the knot values and moving control points, body definition can be changed. Firstly cross-section curves of the body are defined by specifying control points and knots etc.

In polygonalized form, the body is described using a set of small polygons. Body's accuracy is dependent on the number of polygons used to describe it. Mannequin's horizontal cross sections are obtained at varying vertical heights. These cross section curves are approximated with small segmented lines of a constant number. All the vertices of the vertical segments are connected to get a mesh which is an approximation of the body. Selection of the heights, at which cross-sections of the body are taken, depends on the structure of the body. At a portion of the body where the curvature is more, closely spaced cross sections are to be taken, where as less number of cross sections are taken at relatively flat areas of the body.

2.5 Cloth Model

A cloth which is to be draped can be regarded as a woven fabric consisting of horizontal and vertical threads interwoven in a specific fashion. The cloth generally has strong tensile-strain resistance in the thread directions and a weaker shear-strain resistance. Consequently, a cloth deforms its shape chiefly by changing the angles between the horizontal and vertical threads without elongation or shrinkage of its threads. In consideration of this, the following assumptions are made for the cloth model.

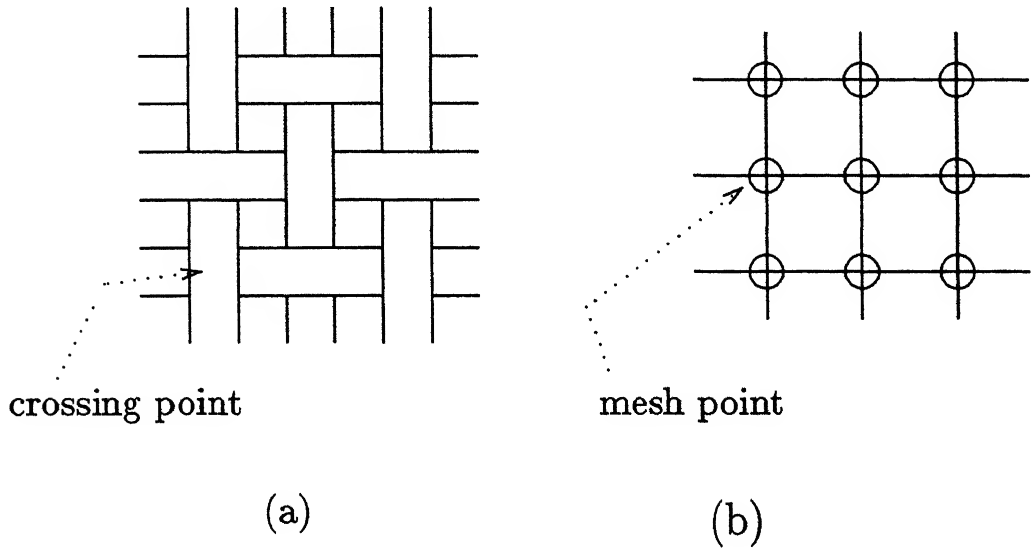


Figure 2.2: a. Yarn Crossings b. Mesh Points

- A piece of cloth is modeled as a piece of woven cloth, in which both the horizontal and vertical threads are inextensible.
- No slippage occurs at a crossing when a cloth is deformed.
- A thread segment between adjacent crossings is straight.

Without loss of generality, the following auxiliary assumptions are made in addition to the above assumptions.

- There is no physical difference between wefts and warps of a cloth.
- The cloth's weaving is a plain weave.

We also assume that the intersection between a weft and a warp defines a crossing called a *mesh point* as shown in Figure 2.2.

Topologically, each mesh point is linked to four adjacent mesh points with equal distances at the outset. Thus the whole cloth can be represented by a linked network of mesh points. Representing every crossing point of a weft and warp is very costly as the number of mesh points in a square of unit distance is of the order of 2000 per sq.cm. So for practical purposes, a cloth can be approximated using a coarser grid of mesh points.

2.6 Geometric Modeling

Three major physical properties of the cloth which are of interest are bending stiffness, shearing stiffness and cloth weight. These physical properties can be mapped into geometric constraints which are to be imposed on the cloth mesh.

2.6.1 Mapping Bending Stiffness

Consider any three connected consecutive points a, b, c in Figure 2.3 which are lying on a horizontal or a vertical line in the mesh. Ideally, when the cloth (mesh) is at rest in a plane, all the three points must be collinear (a, b, c'). The absolute angle (α) made by the line segment joining the points b and c ($\bar{bc}$) with the line segment formed by the points joining the points a and c' ($\bar{ac'}$) is a measure of the bending stiffness of the cloth. Note that $0^0 \leq \alpha \leq 180^0$. The stiffness coefficient of a cloth is directly proportional to the angle α. For a perfectly stiff membrane or metal sheet α is 0^0 while a perfectly non-stiff membrane it is 180^0. Let α be called as *Bending Stiffness Angle.*

2.6.2 Mapping Shearing Stiffness

Now consider any three mesh points such that one point is in horizontal direction (i.e. either towards left or right in the mesh) and the other point is in a vertical direction (i.e. either on upper or lower side) of a central point. (See Figure 2.4)

In the digram q is the central mesh point while p is a point lying towards left of q and r' is lying below q. When the cloth is lying at rest in a plane completely, then the points would have been p,q and r' such that the line segment $\bar{pq}$ is at right angles to the line segment $\bar{qr'}$. The absolute angle (β) made by $\bar{qr}$ with $\bar{qr'}$ is a measure of in-plane stiffness of a cloth. Note that $0^0 \leq \beta \leq 90^0$. A cloth's shearing capability is directly proportional to the value of β.

A perfectly in-plane stiff cloth has a β of 0^0 while a cloth with a very high shear properties has very high (i.e. nearly 90^0) of β. Let β be called as *Shearing Stiffness Angle.*

2.6.3 Mapping Cloth Weight

The cloth weight can be assumed to be a constant downward pull on every node of the mesh. If the weight of the cloth is more, more downward pull is applied at every mesh point of the cloth. Similarly, for lighter fabrics the pull to be applied is less.

2.7 Effect of Geometric Parameters

The drape of the cloth is very much dependent on its physical parameters. Since the physical parameters are mapped into geometric parameters here, there is a direct relationship between the geometric parameters and the draping behaviour of the cloth. The parameters that affect the draping behaviour of a cloth are bending stiffness angle, shear stiffness angle and cloth weight. If the bending stiffness angle is very high, the cloth tends to bend very Easily and gets draped to a larger extent as it can bend through a large angle. If the shearing stiffness angle is very high, then the draping behaviour of the cloth is very high as the cloth can shear with a large angle. If the cloth's weight is more, then the cloth is pulled down by greater force and hence the drape is more.

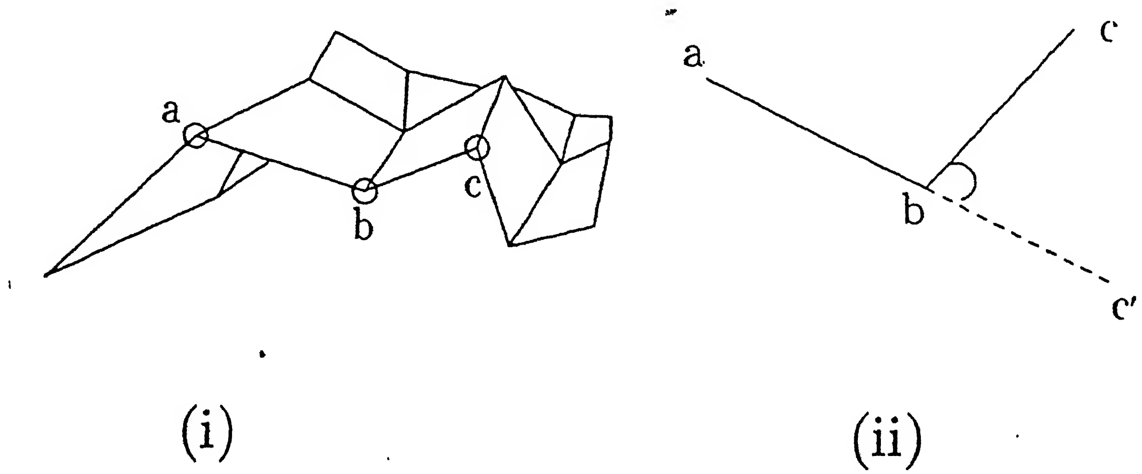


Figure 2.3: Geometric Mapping of Bending Stiffness

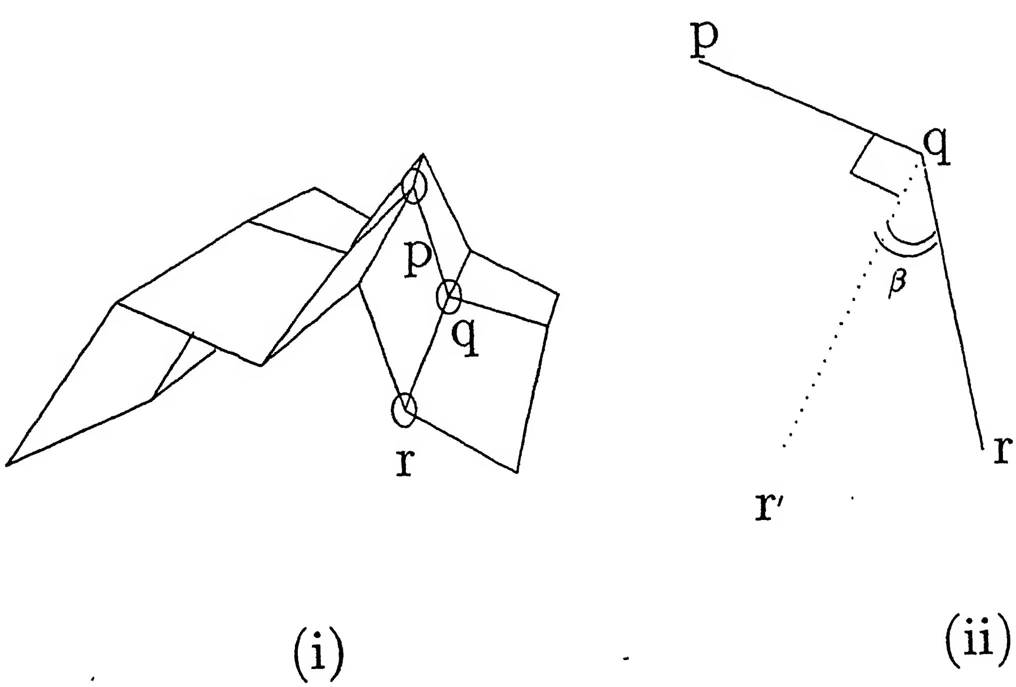


Figure 2.4: Geometric Mapping of Shearing Stiffness

Chapter 3

Graphic Simulation

3.1 Geometric Modeling

First it is necessary to find a way to represent the cloth to be modeled. Here the cloth is assumed to be rectangular and is represented as a grid or two dimensional array of three dimensional points. By increasing the density of the grid, greater resolution of the surface model may be obtained. A very fine grid of cloth, if taken, increases the computational and storage requirements. Moreover final configuration is not much dependent on the fineness of the mesh. A reasonable quadrilateral mesh is taken as an approximation of the cloth.

Mannequin models using both B-splines and quadrangular mesh are developed. But as the collision detection of a cloth with a spline surface is time consuming, a polygonalized mannequin model is considered here. Proper normals are provided for the vertices of the polygons to produce a smooth rendering of the surface of the mannequin. Presently the torso of the mannequin is considered where the legs, hands and the head are absent.

Firstly the cloth is placed on the mannequin so that it is an approximation of a draped cloth. The physical constraints of the cloth to be draped are mapped into geometric constraints of the cloth mesh. Then relaxation is performed on the cloth so that the geometric constraints are enforced on the cloth mesh. Approximating the

cloth, though not mandatory to specify, improves the speed of the relaxation process to reach an equilibrium state.

3.2 Relaxation

Relaxation is the process of moving the points in space so as to satisfy certain given conditions. In our case, the points of the cloth-mesh are to be moved such that the geometric constraints which are in direct relation with the physical constraints of the cloth, are to be met with. The cloth mesh which is properly relaxed, gives us the description of the draped cloth. At the time of relaxing the mesh points, care must be taken to ensure that none of the points will penetrate into the mannequin's body. The crux of the problem is to move the cloth-mesh points in 3D space such that the physical constraints are imposed on the cloth definition while making sure that the cloth does not penetrate into the body.

The geometric constraints are -

- Each mesh point must be unit distance away from all its neighboring connected points. (*Distance relationship*)
- Any of the three connected mesh points either all in a row or a column must not bend at the center point beyond certain angle, which is dependent on the user defined stiffness angle. (*Stiffness relationship*)
- The angle made by any mesh point with any two connected points, out of which one is in the same row and other is in the same column with it, is restricted according to the user defined shear angle. (*Shear relationship*)

One method is to use minimization algorithms for this purpose. But the problem is that the equations to be considered for minimization are a lot in number and minimizing these equations is a difficult task which slows down the relaxation process considerably.

Here, a simple but straight forward method is considered. The relaxation is an iterative procedure. The surface points are to be relaxed such that the shear, stiffness

angles of every mesh point are lying with in the range specified by the user and every mesh point is lying with in a pre-specified distance with every connected mesh point while the gravity force on the cloth is also maintained.

Now, let us consider each of the above constraints separately and see how to relax the points so as to satisfy the constraints.

3.2.1 Maintaining the distance relationship

Consider any four connected points p, a, b, c and d on the cloth mesh. The point p is referred to as *central point* since it is surrounded by four points a, b, c and d as shown in Figure 3.1. Suppose that the central point is at a distance of d_1, d_2, d_3 and d_4 from a, b, c and d respectively. Let the unit distance between p and any other connected point be d_{unit}. So, in order to maintain a distance d_{unit} from a, p has to move through a distance of $d_{unit} - d_1$ along the line joining both p and a. The direction of the movement is away from a.

Let us denote the vector quantity of this distance to be moved by $\mathcal{D}_1$. Similarly, we can get $\mathcal{D}_2$, $\mathcal{D}_3$ and $\mathcal{D}_4$ corresponding to the points b, c and d respectively. Net sum of $\mathcal{D}_1$, $\mathcal{D}_2$, $\mathcal{D}_3$ and $\mathcal{D}_4$ together gives us the vector in which direction p has to move in order to maintain equi-distance relationship with the surrounding connected points. Let this vector sum be called as *Net Displacement Vector* ($\mathcal{D}$).

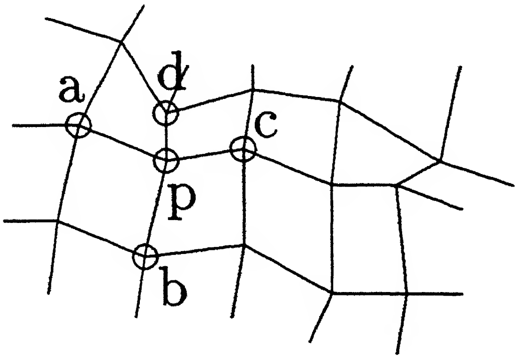
Let the co-ordinates of p are (x_1, y_1, z_1) and that of a are (x_2, y_2, z_2). Then the direction vector of $\bar{pa}$ (i.e. line joining p and a) is given by $(x_1 - x_2, y_1 - y_2, z_1 - z_2)$. Let the distance between them be denoted by dis and is given by

$$dis = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Then a point p' (x', y', z') at a *unit_distance* from a along the direction of $\bar{ap}$ is given by

$$\begin{aligned} x' &= x_2 + \left(\tfrac{unit\_distance}{dis}\right) \cdot (x_1 - x_2) \\ y' &= y_2 + \left(\tfrac{unit\_distance}{dis}\right) \cdot (y_1 - y_2) \\ z' &= z_2 + \left(\tfrac{unit\_distance}{dis}\right) \cdot (z_1 - z_2) \end{aligned}$$

So the additional distance to be moved by p is $\mathcal{D}_1$ and its value is



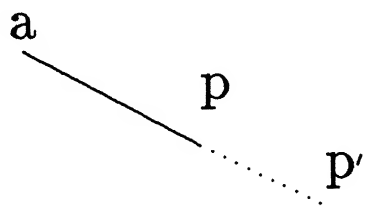


Figure 3.1: Maintaining Distance Relationship

$(x' - x_1)i + (y' - y_1)j + (z' - z_1)k$

i.e.

$(1 - \frac{unit\_distance}{dis}) \cdot (x_2 - x_1)$

Note that every mesh point may not have four surrounding points. So for the mesh points on the sides of the cloth, net displacement vector is the sum of the Displacement vectors for each connected points.

Here some tolerance(ϵ) is allowed for distance checking. Above expression for $\mathcal{D}_1$ will be a zero vector if *dis* falls within the tolerance range. (i.e. $unit\_distance - \epsilon \leq dis \leq unit\_distance + \epsilon$ where ϵ is a small positive real number). This process of giving tolerance is valid because whenever a cloth gets sheared, its length or width gets changed and this tolerance can be assumed to compensate this length change.

3.2.2 Maintaining the stiffness relationship

Now consider any three consecutive connected points in the mesh such that all of them are either in the warp direction or in the weft direction. Let these points be denoted as p, q and r, as shown in Figure 3.2 . Consider a point r' on the extended line of $\bar{pq}$.

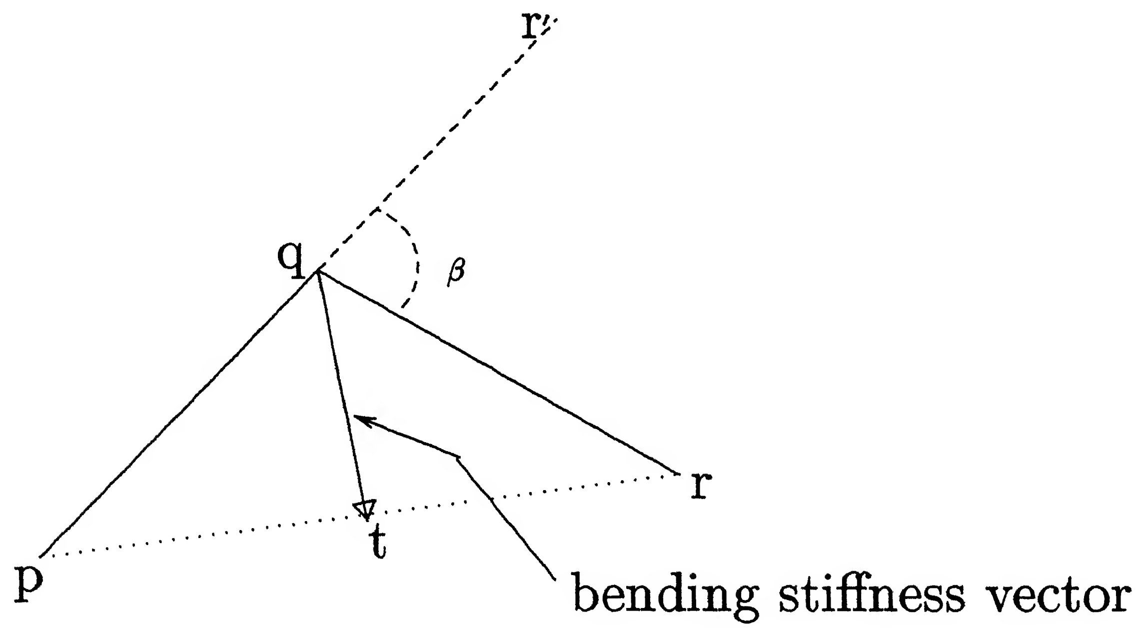


Figure 3.2: Maintaining Bending Stiffness

The angle $\angle r'qr$ denotes the bending angle (β) of the mesh at q. If this bending angle is more than the bending stiffness angle(ϕ) mentioned by the user, then in order to acquire a less bending angle, point q tries to move towards the line $\bar{pr}$.

One simplest method is to move the point towards the center of the line segment $\bar{pr}$. Let the center be t. So the stiffness vector associated is the difference of the position vector of t to the position vector of q (i.e. $\vec{qt}$)

Note that for every inner mesh point, there are two such vectors. One for maintaining the bending angle in weft direction (γ_1) and the other for maintaining in warp direction (γ_2). Let the sum of these two vectors be denoted by γ which is called as *Net Stiffness Vector.*

It can be observed that, for a point which is lying at the edge of the mesh, net stiffness vector may be composed of a single vector or none. For ex. for a point on the extreme left end of the mesh, we can not find a stiffness vector in the row direction. For the points at the four corners of the cloth, net stiffness vector is a null vector as we can not define a stiffness vector in both row and column directions.

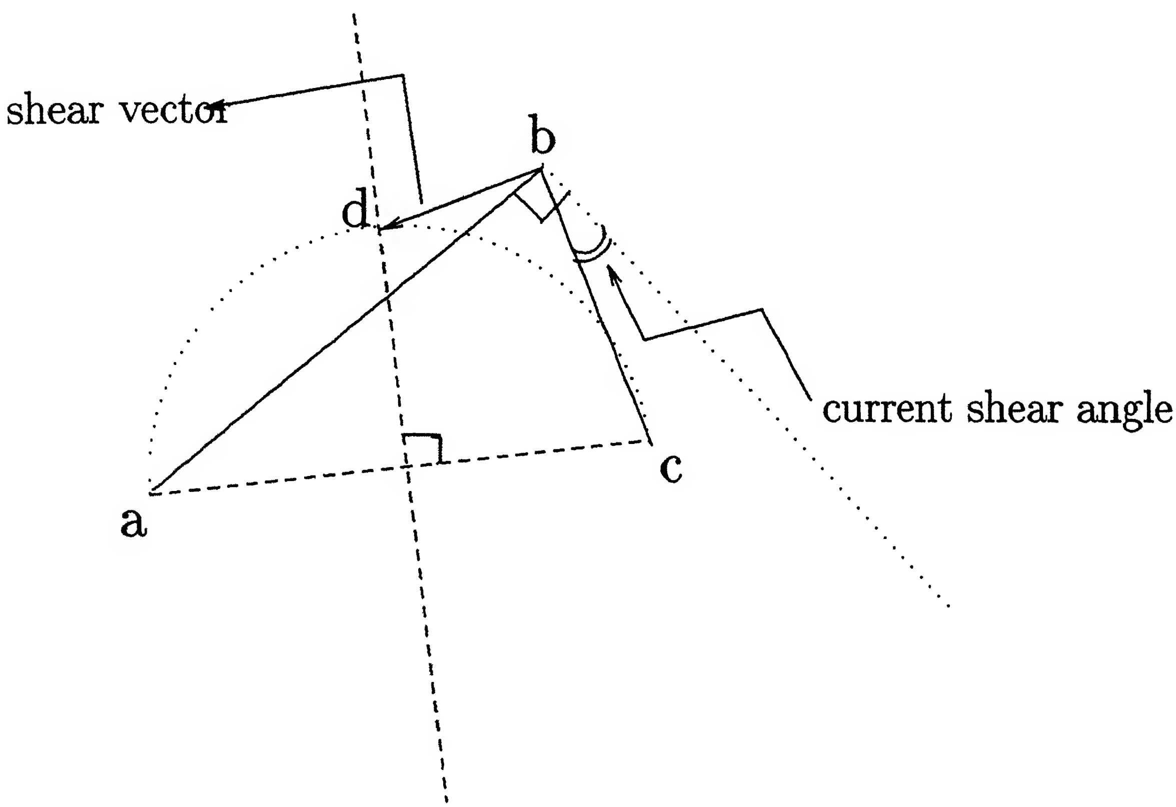


Figure 3.3: Maintaining Shear

3.2.3 Maintaining the shear relationship

Let b be a point in the interior of the mesh. Consider any two points a and c such that they are connected to b where a is immediately clockwise or anti-clockwise in direction with c. (See Figure 3.3)

Deviation of the angle $\angle abc$ from 90 degrees is a measure of the shearing stiffness of point a, b, and c at the point b. This deviation is our shearing angle. If this shearing angle exceeds the user defined *Shear Stiffness Angle*, then the point b has to move to a new point b' such that the shearing angle of b' made with a and c is below shear stiffness angle.

One simplest way of doing this is to move b to a point which is on the bisector of $\bar{ac}$ and makes a right angle with the line $\bar{ac}$. Let the destination point be d. Our shear vector is the the vector originating from b and joining d ($\vec{bd}$).

Note that there are four such vectors for every inner point of the mesh. Sum of the four shearing vectors is our *Net Shearing Vector.* For the points at the ends of the mesh, this vector may be composed of three or two shearing vectors.

For maintaining the weight factor of the cloth, we add a vector at every mesh point such that it is pointing downwards. The magnitude of this vector must be a suitable one.

Now, for maintaining the above relationships, a central point has to be moved along a vector which is obtained by adding net-displacement vector, net-stiffness vector, net-shear vector and weight-density vector together. The amounts of above vectors to be summed up to get our net-resultant vector is an important consideration. For simplicity, we can add equal amounts of various vectors to get resultant vector through which we move a present point. Care must be taken such that the destination is always above the body of the mannequin. If the point is penetrating into the body, we must move the point to the nearest possible points to the body.

3.3 Refinements in Relaxation

Even though above procedure looks to be fine, there are some problems with it. The first problem arises of the fact that we are ignoring the fact that some vectors have to be given due importance. For ex. there may be a case, where the point's stiffness and shear angles are considerably close to the user defined angles and the displacement to be moved by the point is more. We neglecting the effect of a dominating vector. Other problem is that we are ignoring the magnitude of individual vector in the summation process. Any vector's magnitude has to be dependent on the deviation of its parameters from the user-defined parameters. Moreover, we should try to maintain equi-distant relationship with the connected neighbors of consideration.

The following example makes it clear.

Consider that the current stiff angle is slightly greater than the maximum stiffness angle. (See Figure 3.4) Instead of moving the point towards the line $\bar{pr}$ so as to maintain both bending stiffness vector and distance, we are trying to move our point

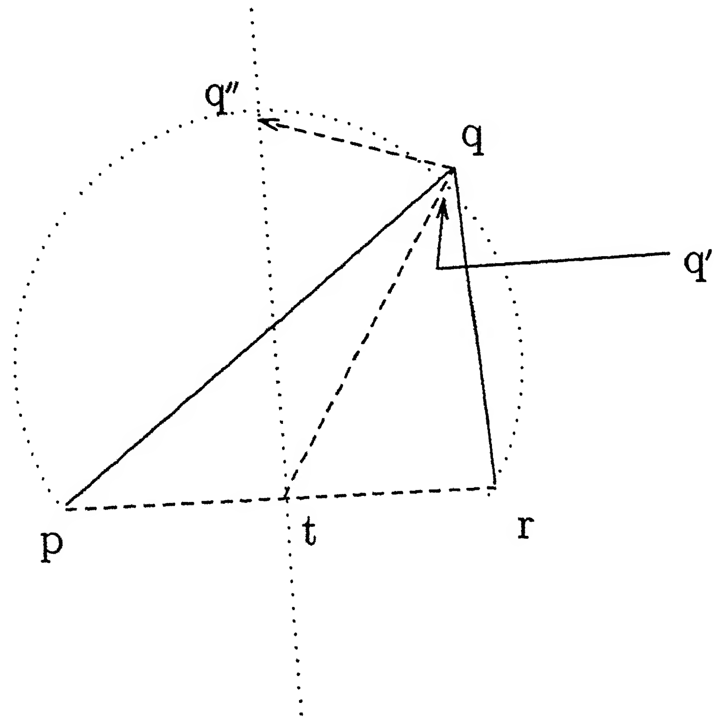


Figure 3.4: Inaccurate stiff vector

q to the center of the midpoint of the line $\bar{pr}$ (i.e.s) because of which our stiff vector is very high in magnitude.

Instead, we can move the point q to q'' such that q'' is on the bisector of the line $\bar{pr}$ and $\angle pq''r$ is 180-maximum_stiff_angle.

Similarly, at the time of moving a central point so as to maintain a user defined maximum shearing angle, we move the central point to a point where the angle is 90 $\pm$ *maximum_shear_angle*. If our current shear angle is greater than 90 degrees, then the central point is moved to make an angle of 90+maximum_shear_angle. Otherwise the point has to be moved where the angle is 90-maximum_shear_angle.

To improve further, we will try to move a point depending on the magnitude of a vector. We may have to scale our stiffness, shearing and displacement vectors depending on the bending stiffness angle, shear stiffness angle and displacement of a point.

One crucial issue is how to get the net-result-vector by adding appropriate amounts of net-displacement vector, net-shearing vector, net-shear vector and weight density

vector. We have to evolve a relation between the amounts of above vectors to be added. Since we are dealing with distance in one case while dealing with angles in other cases, there has to be a correlation between the angle which has to changed and the distances to be moved. One way of doing this is to relate the maximum allowable distance to be moved by a point to the maximum allowable shear & stiffness angles.

So the magnitude of the stiffness vector, when the current stiffness angle is greater than maximum stiffness angle, is given by

$$unit\_distance \cdot \frac{(maximum\ stiffness\ angle - current\ stiffness\ angle)}{180}$$

If the current stiffness angle is less than maximum stiffness angle, then the magnitude of the stiffness vector is zero.

Similarly the magnitude of the shear vector, when current shear angle is greater than maximum shear angle is given by

$$unit\_distance \cdot \frac{(maximum\ shear\ angle - current\ shear\ angle)}{90}$$

Relaxation process is an iterative procedure here. For every mesh node, we find a net resultant vector according to the above strategy and the points are relaxed. We have to repeat this procedure until the maximum magnitude of net vector falls below a certain tolerance value.

At the time of relaxation, we can relax one point after another or all the points at a go. The problem with the earlier method is that one point's relaxation affects the further calculations of the next points to be relaxed. More over it is costly in terms of the number of iterations required to reach an equilibrium and most crucial is that our final configuration is mostly dependent on the order in which the points are relaxed. So it is advisable to calculate the net resultant vectors for all the points and then relax all the points.

3.4 Preventing Body-Cloth Intersections

During the relaxation process, we have to make sure that the cloth does not penetrate into the body. Firstly we have to find out whether the cloth is intersecting with the mannequin's body or not. This problem can be solved using two methodologies.

- Surface-Surface intersection
- Surface-Point intersection

In the first method, every polygon of the cloth has to be checked for intersection with every polygon of the mannequin. This method is difficult as we are required to solve the plane equations and have to make checks to find out the intersections.

In the second method, every mesh point of the cloth is checked with all the polygons of the mannequin. If all the polygons of the mannequin are defined in an order, then defining outside and inside of the mannequin becomes simpler. By checking the side of a polygon of the body in which the point lies, we can tell whether the point is lying outside the body or not. Bounding box checks can be used to reduce the number of the polygons to be checked. But the problem is that we can not make sure that a cloth's polygon is intersecting with the body or not. If the body is having sharp protrusions, all the mesh points may lie above the body while there is still intersection with the cloth. This problem can be circumvented by defining the mannequin in such a way that there are no sharp protrusions or by increasing the minimum distance that is to be maintained between the cloth and the body.

If the minimum distance to be maintained between the cloth and the body is defined, then whenever the mesh point comes very close to the body, certain penalty can be given by pushing it up through some distance. This *penalty method* ensures that there is no intersection between the body and the cloth.

3.5 Self Intersection of Cloth

Some times the cloth surface may intersect itself. This self-intersection is difficult to be detect. One way is to apply a repulsive force associated with the cloth surface. Whenever the mesh point comes close to the cloth surface, this repulsive force becomes very high that the movement is prohibited. This method is very costly as we have to carry out distance checks with every polygon of the body.

Other method is very simple. When ever a mesh point is being moved, make sure that it's coordinates fall with in the infinite 3D solid whose edges the neighbours of the current point lie. This method does not allow certain non-intersecting configurations.

Chapter 4

Rendering

Once the surface of the cloth is relaxed, it can be converted into a polygonalized surface, ready for any of the further rendering techniques. Each polygon can be rendered separately by calculating its normal. Rendering the surface in this fashion may appear realistic for some materials like metal sheets but, in general, the surface will lack a cloth-like quality. A cloth texture may be mapped on to the surface, but the translucent effect of the cloth still may not be achieved because of this. The most effective way of rendering a cloth is actually to render each and every thread individually. This is certainly computationally more expensive when compared to the above said methods, but this kind of detail allows realistic close-ups of the cloth.

In order to achieve a surface which does not look like a fish net, a very fine mesh of lines must be fit to the surface. However, the surface approximation and relaxation stages become computationally intensive when run on a fine grid of points. Furthermore, there is not much difference between the overall structure of a surface calculated on a very fine grid from that calculated from a coarser grid. Therefore, the relaxation method is run on a coarser grid and the remaining points are filled in by fitting splines to the calculated grid coordinates. A finer mesh is created by first fitting the splines to the grid points along each column of the grid. Corresponding points along each of these splines are used as the knots for the splines to be fit along the rows. (See fig 4.1)

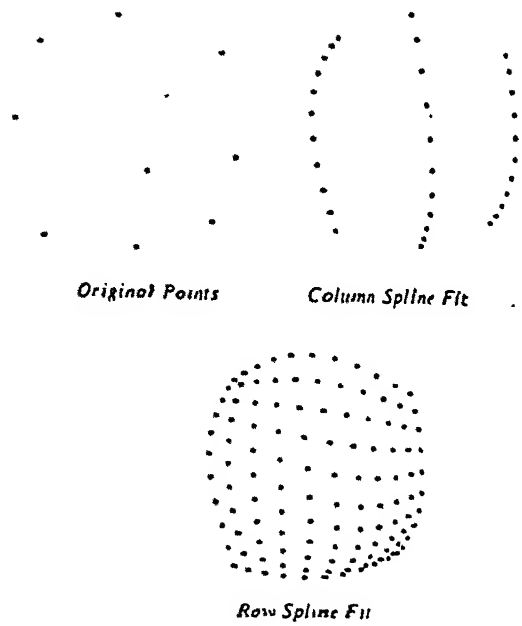


Figure 4.1: Fitting Spline Points in a Mesh

The rendering technique used here treats the cloth as a collection of line segments whose end points are connected in the form of a quadrilateral mesh. Every line segment is treated as a cylinder to give a 3D effect to the cloth. Assuming a rectangular view volume, rays are cast on the cloth perpendicular to the viewing plane. While checking for the intersection of a ray with a line segment, a tolerance is allowed. Because of this tolerance, a ray, whose distance from the line segment is less than the tolerance, cuts a a 3D cylinder around the line segment, whose radius is equal to the tolerance. By varying the tolerance, the thickness of the cloth can be altered. As the calculation of the distances between the line segment and the ray are costlier, an approximation can be made. Instead of calculating the distance to the line, we can take horizontal or the vertical distance with the line depending on the slope of the line. Though this approximation changes the radius of the cylinder by a factor of $\sqrt{2}$ the effect of it is hardly noticeable.

Three dimensional effect can be obtained by perturbing the normals of the line segments. For every line segment, three normals are considered of which one is perpendicular to the projection plane while the remaining are two normal vectors to the line, which is the projection of the 3D line segment in the viewing plane.

Chapter 5

Fitting a Woven Cloth On a Doubly Curved Surface

5.1 Introduction

Fitting of a cloth on any surface is a process of deforming the cloth and applying it to the surface so that it is in contact with the surface without any gaps and wrinkles. Woven-cloth composites, which are one of the major composite production forms, are being extensively used for reinforcing structural elements in the aircrafts. Firstly 'plies' of composites are cut from the rolled sheets of plain woven cloth or impregnated cloth. Then they are pressed and fitted to moulds to form 3D composite parts. Presently, there are very few design-automation tools for fitting process and the manufacturing process is mostly carried out by hand which is very much time consuming and inefficient. More over fitting done by hand requires a lot of effort. In order to overcome these overheads, there is a great need for automating the fitting process (i.e. finding the 3D ply outlines and automatically creating the 2D flattened form).

Deformation of a broad cloth composite is a complex function of the given material of the ply, the given 3D surface shape and the initial conditions for the fitting. A general frame work for predicting a variety of fittings on the basis of different initial

conditions is proposed by Aono, Wozny et al [1]. An outline of the mapping algorithms is discussed here.

A broad cloth composite ply can be modeled as a set of extensible vertical and horizontal threads and a 3D curved surface as a nonuniform rational B-spline (NURB) surface. With the assumption of inextensibility, the ply is mainly deformed by changing shear angle between the horizontal and vertical threads of the ply.

In the fitting process, firstly the initial conditions are specified using which a line in a 2D ply and its mapped curve on the 3D surface. Then all the remaining points of the ply are mapped on to the 3D surface by using some mapping algorithms.

5.2 Initial Conditions

Specifying initial conditions is the process of fixing an initial line in the cloth ply in 2D mesh and fixing a curve on the 3D surface which is the mapping of the initial line in 2D. Initial conditions must be sufficient but not excessive for the mapping between a point on a piece of cloth in 2D space and the corresponding point on a surface in 3D space to be uniquely determined. If they are overspecified, it is likely that no solution will satisfy given initial conditions. If, on the other hand they are underspecified, a unique solution may not be found. Initial path specifications are very important as they determine the flexibility of the mapping problem. There are various methods for specifying them.

5.2.1 Method-1

In this method, initial conditions are specified by fixing of two yarn paths both in 2D and 3D spaces. These paths are called 'constrained yarn paths'. In 2D-space one weft yarn and one warp yarn are selected as a pair of constrained yarns. In 3D space each constrained yarn path is mapped into a curve that is defined by a sequence of equidistant points on the given surface. By specifying a pair of constrained paths, both the 2D cloth region and 3D surface region are generally divided into four regions. The mapping calculation of points between 2D and 3D spaces is carried out

independently for each quadrant. Though this method is straight forward and simple, it has some problems inherent to it. First problem is that it does not allow sufficient flexibility in specifying the constrained yarn paths in 2D space as it allows only the paths to lie on perpendicular warp and weft threads. Second problem is that the cumbersome task of calculating the points of equidistance on the 3D NURBS surface on which the cloth is to be fitted is left to the user. Third problem is that there is a possibility of leaving uncalculated regions within the original surface area when the surface has concave shape or when the constrained yarn paths are badly selected.

5.2.2 Method-2

This method which was originally suggested by Aono, Wozny et al [1] removes the earlier two problems.

Initial Conditions in 2D space

This method of specifying initial conditions in 2D space is based on the actual manufacturing practice used during the lamination process, where one arbitrary path is fixed that is not necessarily aligned with a particular thread and then the ply is swept in arbitrary directions. One initial path is defined instead of defining two perpendicular initial paths. Let this path be called as *base path* or *guideline*. When a base path is defined, the 2D space gets divided into two regions. The base path intersects with the warp and weft mesh points to give *auxiliary* mesh points. Then two sweep directions are to be chosen to sweep the ply. Previous method is a specialization of this method, where the base path coincides with a specific thread and the sweep directions are perpendicular to the base path.

Initial Conditions in 3D space

The result of fitting a ply depends on the surface to be fitted also. To automatically map the mesh points on the 3D surface, without calculating them by hand, a base path is to be defined on the 3D surface. Instead of defining the base path along a

fixed directions or by using parametric equations, the base path is defined using a base plane whose intersection with the surface results in the base path.

5.3 Mapping Process

The problem of mapping the mesh points on to the 3D surface is simplified as a problem of surface-surface intersections. There are two stages in the mapping of the mesh points of which one is regarding the mapping of auxiliary mesh points and the other is regarding the mapping of regular mesh points.

5.3.1 Mapping Auxiliary Mesh points

The process of this mapping starts with a known on the base path of the 3D surface. It is assumed that the distances between the auxiliary mesh points in 2D ply are preserved on the base path of the 3D surface also. A point which is at a distance d from the known point x on the base path is found as the intersection of a sphere, whose origin is at x and has a radius of d, the base plane and the 3D surface which is described as a NURB surface.

Suppose the co-ordinates of the known point or start point is (x_0, y_0, z_0) and the direction vector of the base plane is (x_n, y_n, z_n).

Then the equation of the plane is

$$x_n \cdot (x - x_0) \; + \; y_n \cdot (y - y_0) \; + \; z_n \cdot (z - z_0) \;\; = \;\; 0 \tag{5.1}$$

Similarly the equation of the sphere is

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 \; = \; d^2 \tag{5.2}$$

Let the equation of the 3D surface be

$$r(u, v) = \frac{\sum_{i=0}^{C_u-1} \sum_{j=0}^{C_v-1} w_{i,j} \cdot C_{i,j} \cdot N_i^m(u) \cdot N_j^n(v)}{\sum_{i=0}^{C_u-1} \sum_{j=0}^{C_v-1} w_{i,j} \cdot N_i^m(u) \cdot N_j^n(v)} \tag{5.3}$$

where $C_{i,j}$ is a control point of the surface.

Since the coordinates of a point on the surface $(x(u,v),\ y(u,v),\ z(u,v))$ the required point is found by substituting x, y, z values in the above two equations.

Newton Raphson method can be used to find out the points on the base path.

5.3.2 Mapping Regular Mesh points

Regular points are of two types. One type are the ones which are computed based on two known points and the other type are the ones computed based on a single known point.

Firstly, the auxiliary points are used to find the nearest regular points to the base path. Later on these regular points are used for finding subsequent regular points.

In the first case, the problem of finding a point is reduced to finding an intersection of the surface and two spheres whose radii are the warp and weft distances of the destination point. The second case is discussed further below. At the time of mapping a point, if the newly mapped point makes a shear angle with connected points is out of the range of the ply then the mapping process is stopped as the cloth ply can not be fitted on to the 3D surface.

5.4 Scanning Algorithms

Scanning algorithms describe the order in which the mesh points are mapped on to the 3D surface. The final sweep obtained is very much dependent on the order in which the points are mapped. The mapping algorithms are applied to each mesh in the 2D ply, where as the scanning algorithms are applied to a set of mesh points.

Consider an example shown in Figure 5.1, in which regular mesh points are given by a 4 x 4 network of nodes labeled from 1 to 16, auxiliary points of te base path are given by a sequence of nodes labeled from a to f and the sweeping directions are supposed to be perpendicular to the base path.

Figure 5.2 shows the scanning order in a directed acyclic graph (DAG) which is referred to as *dependency graph* of the mesh points of 2D ply. In the dependency graph, the nodes denote the mesh points while the edges represent the dependency

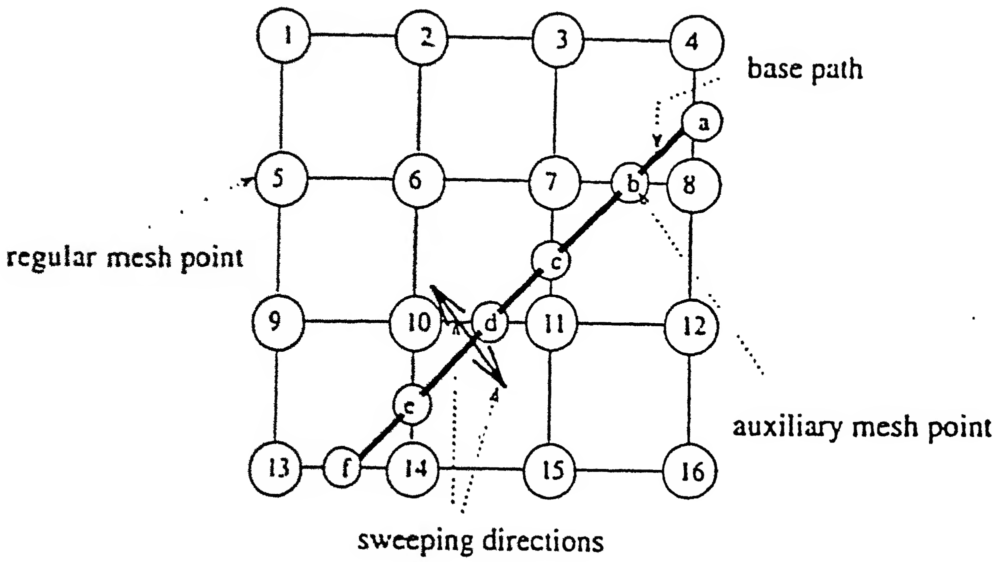


Figure 5.1: A 4x4 Regular Mesh of Points

relationship between the mesh points. Once the mesh points at a level are mapped, the points at the next level of DAG, to which arcs are pointing from the previous points, are mapped. In general, there are two sorts of nodes in DAG. They are *indegree-one nodes* and *indegree-two nodes*. Indegree-one nodes require special attention as they are the source of uncalculated surface regions. Mapping these points is difficult as there is only a single known point.

Mapping indegree-one nodes

Indegree-one nodes require a special mapping algorithm. It is impossible to determine the mapping for them uniquely as they have a single known adjacent mesh point. One huristic is described here which assumes that the local geometry is preserved around the indegree-one node. More over, it is assumed that the angular relationship between the thread and the base path at a mesh point in 2D space is preserved on the tangent plane at the mapped point in 3D space.

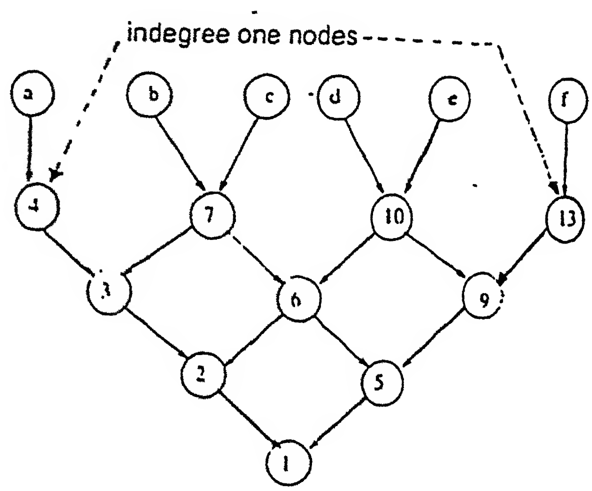


Figure 5.2: Directed Acyclic Graph

Scanning Mesh Points

Once the auxiliary mesh points are mapped on to the 3D surface, the regular mesh points are to mapped. The exact order of them is determined by their distance from the base path. The nodes that are connected to the auxiliary mesh points are to mapped first. There are two sets of such points, which are on both sides of the base path. These sets are called *frontiers*. In the Figure 5.1, two frontiers are { 4, 7, 10, 13 } and { 8, 11, 14 }. Once all the points of one frontier are mapped, next frontier has to be found out. This process goes on until we get a single point in the frontier and this point is mapped.

Chapter 6

System Design

6.1 System Features

In this chapter various features of the system and the design issues are considered. The cloth draping system is very much helpful for predicting the drape of a fabric on the mannequins. An apparel designer can specify various physical properties of the fabric and can see the drape of the cloth after relaxing the cloth's definition.

Main quality attributes that the system possesses are

- User Friendliness
- Portability
- Interactivity
- Modifiability
- Manageability

A good user interface is provided by the system. The system is highly interactive. The user can specify the physical properties of the cloth and can see the draping effects by relaxing the cloth mesh. The user can choose a particular rendering and display method for the cloth too. The system is easily portable. Though the program is currently written in *Starbase* for TSRX work-stations of HP, all the machine

specific graphic routines are separately maintained in a file. Just by modifying these routines, the system can be easily ported to other work-stations. The program itself is modifiable and well commented. All the constants are declared using *#define* s which enables us to modify the constants as and when required. The whole system is partitioned into different modules with different functionalities. Each module can be easily modified without affecting the whole system, thus the system is easily manageable.

6.2 System Design

The design of the system is very much flexible and can be modified easily. As an example, the cloth's initial approximation or the mannequin model can be replaced with newer ones. The system is divided into different modules according to functionality. These modules are easily modifiable and some of them can be used for other systems like the ones which are meant for draping curtains etc.

Basic modules of the cloth draping system are

- User Interface Module
- Relaxation Module
- Cloth Description Module
- Mannequin Description Module

6.2.1 User Interface Module

User interface module forms the front end portion of the system. It provides the user with an interface. The user interacts with the system through this interface. Various options are provided to specify the physical property constants, to choose different rendering methods etc. This module inturn communicates with all other modules and gives the output to the user. User can choose options by clicking appropriate options in the menu and can input the physical property constants using the key board.

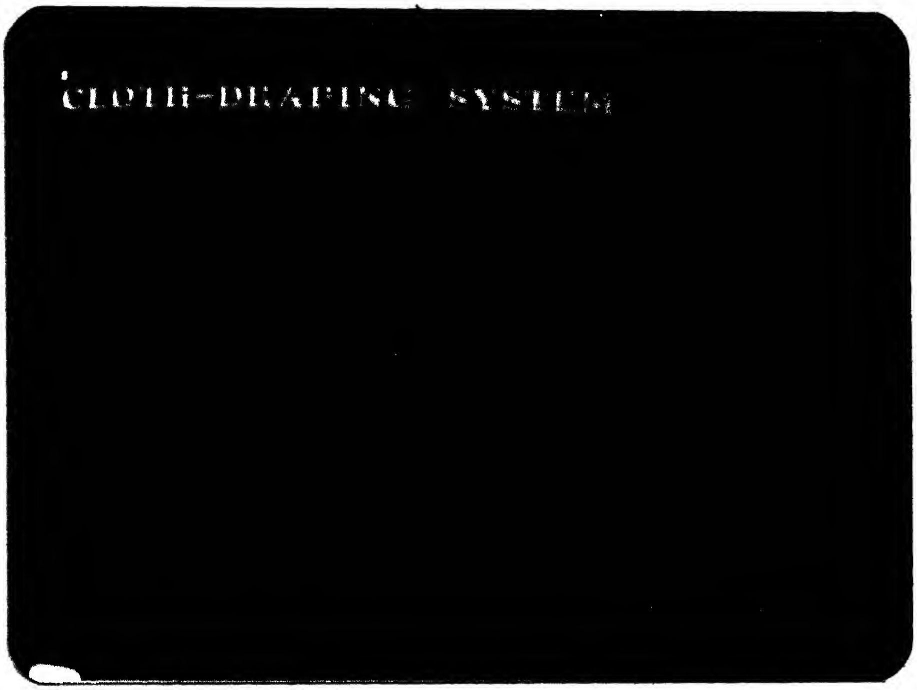


Figure 6.1: User Interface

In this module, a function for displaying generalized menu displays is given. This function can be used to generate any number of menus. Options in the menus can be easily changed by modifying or adding C-structures of the current system.

A sample menu of the user interface is shown in Figure 6.1.

6.2.2 Relaxation Module

Relaxation module forms the heart of the whole system. As the name suggests, it describes the relaxation procedure. This module has various routines for calculating displacement, bending stiffness and shearing stiffness vectors. This module can be easily used in other cloth draping systems which are meant for seeing the drape of curtains and table cloths etc.

Relaxation routine is called for every mesh point which calls other routines which compute the displacement, bending and shearing stiffness vectors. For every mesh point, all its connected neighbouring points are to mentioned for calculating the above vectors. User can specify the number of iteration of relaxation is to be performed on the cloth mesh and so the relaxation procedure has to be carried out iteratively for

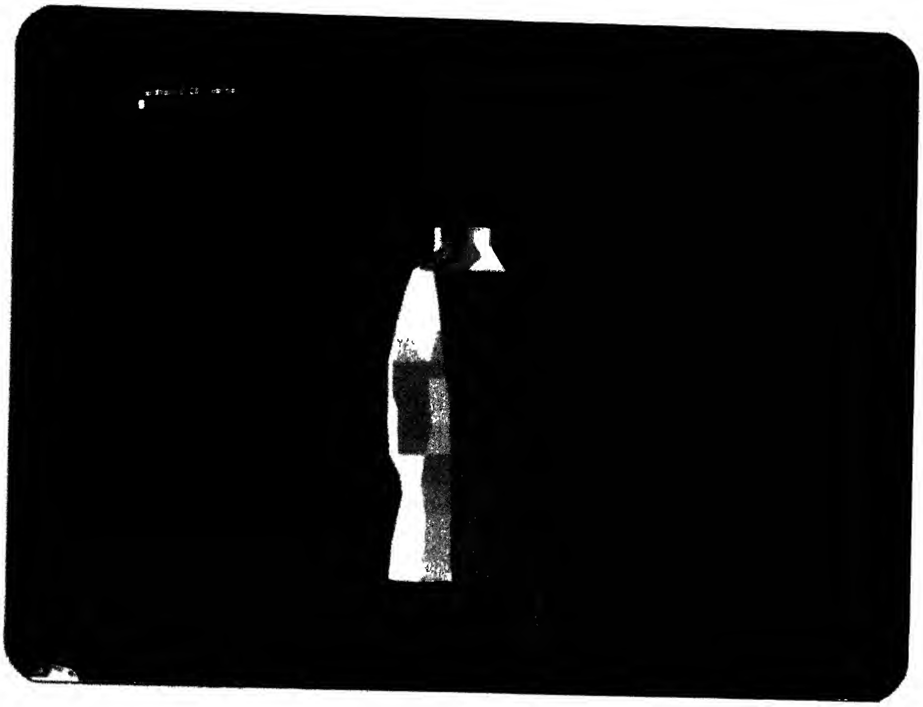


Figure 6.2: Cloth with Rendered Yarns

the number of user specified iterations.

6.2.3 Cloth Description Module

This module has the description of the fabric to be draped. The description of a fabric comprises of its definition and the rendering information.

The cloth's initial approximation for the relaxation process can be easily modified by the user. Even though this approximation is not mandatory, it reduces the burden of relaxation process considerably. Cloth's definition is provided as a series of three dimensional coordinates of the mesh points. Cloth's definition gets altered whenever relaxation is performed on the mesh.

Rendering part of the cloth also falls in this module. User can choose any one of the rendering methods that are presently available or can choose the mesh model. Currently two rendering methods are provided to the user. In the first method, individual yarns are approximated from the cloth mesh and each and every yarn is rendered as a cylinder to give translucent effect to the cloth. In other method, normals of the cloth are calculated at every node of the mesh and are used for rendering the

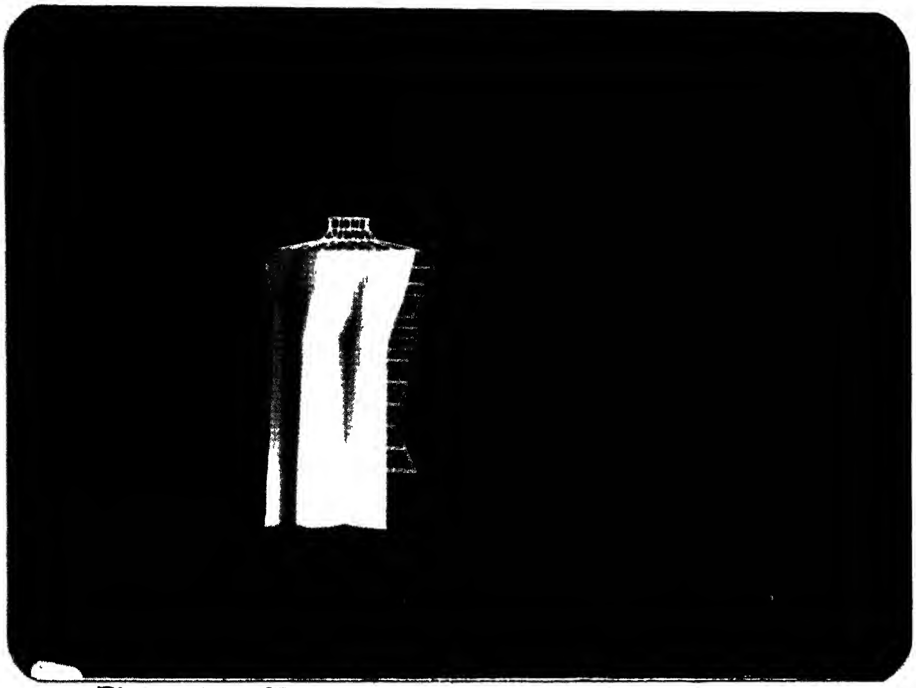


Figure 6.3: Cloth Rendered by Computing Normals

cloth with Gourad shading model. Texture mapping is also done in this method to give good results.

Figure 6.2 shows a cloth piece whose individual yarns are rendered as a cylinder. Figure 6.3 shows a piece of cloth which is rendered by calculating normals at every mesh point and by mapping texture onto it.

Self-intersection prevention of the cloth is taken care of in this module. Presently self-intersection of the cloth is removed by making sure that the relaxed point does not lie with in the infinite quadrangular solid which is passing through the four neighbouring points and the points are lying on the infinite edges of the solid.

6.2.4 Mannequin Description Module

This module has the description of the mannequin model. Currently, a polygonalized model is provided in the system. The mannequin is described by defining polygons and then providing normals at the vertices of the polygons to give smooth shading effects.

Checking whether a cloth-mesh point is penetrating into the body is incorporated

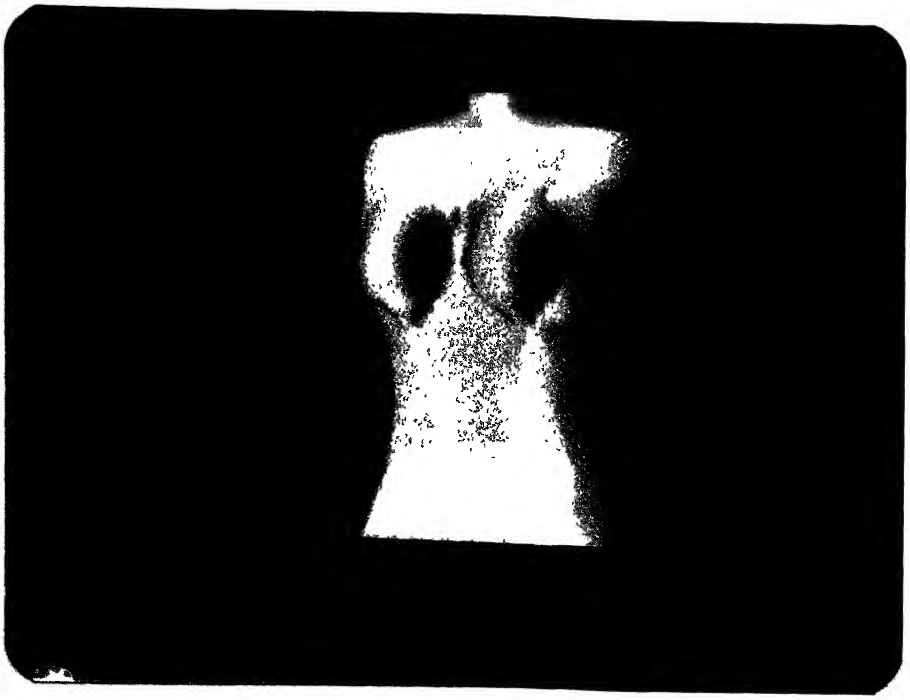


Figure 6.4: A Mannequin

in this module. This is because, this checking relates to the definition of the polygons of the mannequin. Currently, polygon-point intersection method is used for preventing the penetration of the cloth into the mannequin.

A mannequin model used by the system is shown in Figure 6.4.

6.3 Examples

Different types of fabrics are simulated by specifying different physical properties. A stiff cloth which resists bending is simulated by specifying high bending resistance. For specifying high bending resistance, a very less stiffness angle is specified.(See Figure 6.5).

A cloth with moderate shear resistance is shown in Figure 6.6.

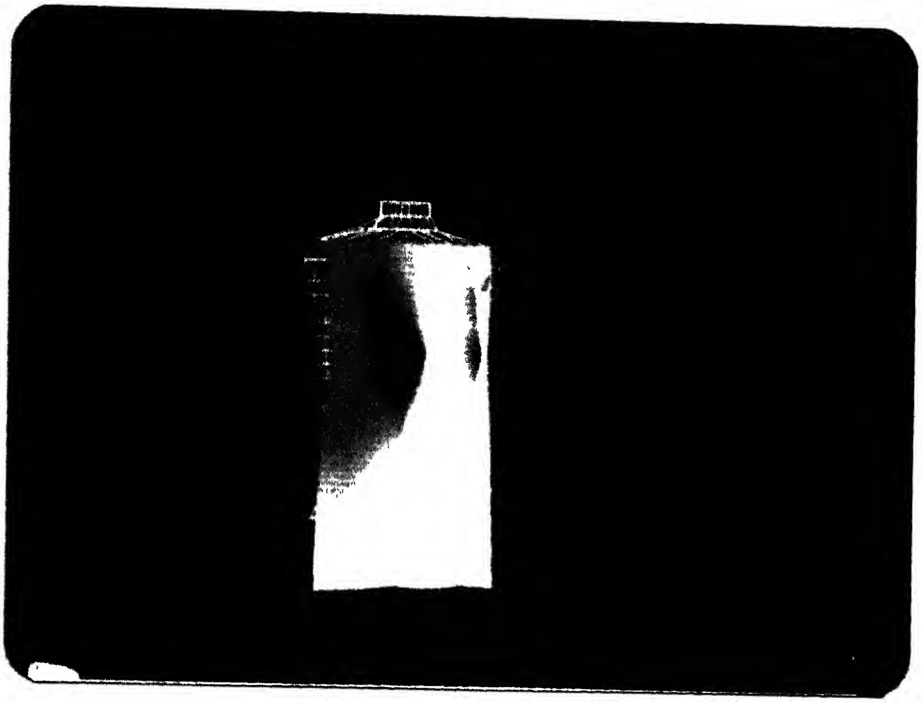


Figure 6.5: Stiff Cloth

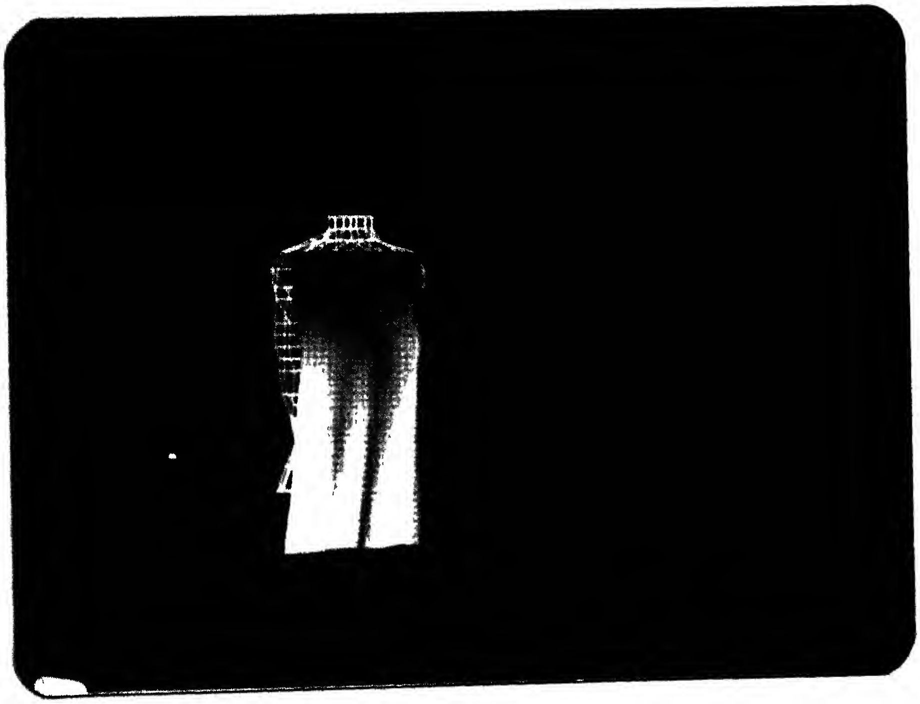


Figure 6.6: Moderate Cloth

Chapter 7

Conclusions

7.1 Summary

Drape of a cloth is of considerable importance to the apparel designer. Many researches are going on to simulate the cloth's drape which are useful for a fashion or apparel designer. A methodology is presented which mainly aims at the user who wants to simulate cloth-drape of different fabrics having different physical properties on mannequins.

Present methodology maps the cloth's physical properties into geometric constraints. A cloth is approximated as a quadrilateral mesh and these geometric constraints are imposed on the points of the mesh. The mesh points are relaxed according to the geometric constraints and the final configuration gives the approximate drape of the cloth. Currently the physical properties considered are bending, shearing and weight of a cloth. The relaxed mesh is used to render the cloth effectively. Present implementation is done on TSRX work-stations of HP 98731 series. Graphic Simulation is done using STARBASE graphic library. An user interface is provided for specifying cloth weight, bending stiffness and shear stiffness of a cloth. An outline of fitting a cloth on a doubly curved surface is also described.

7.2 Further Extensions

There is a lot of scope for extending the present work. The only physical properties that are considered are bending stiffness, shearing stiffness and cloth weight. Effect of various other physical properties like thickness of the cloth, yarn density etc can be considered to give more realistic drapes of the cloth materials. The current model can be extended to estimate the draping behavior of stitched garments. Friction between the body and the cloth and tension in the cloth membrane can also be incorporated easily into the current model and can be tested.

Current mannequin model can be replaced by different mannequins of varying sizes. Mannequins in parametric form can be incorporated in the system. Present rectangular piece of cloth can be replaced by garments which are more realistic. Surface-surface intersection methodology can be adapted to check the intersection of the fabric with mannequin.

Rendering needs considerable improvement. Shadows and animation of the cloth can also be incorporated into the system. Various texture mapping mechanisms and shading models can also be tried. There is also a need to consider the actual photographic evidence to compare with the experimental results.

Various texture mapping mechanisms and shading models can also be tried.

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Appendix A

Point Location

Suppose a (x_1, y_1, z_1), b (x_2, y_2, z_2) and c (x_3, y_3, z_3) are three points in space. We are required to find a point p (x', y', z') such that it lies in the plane passing through a, b and c (Σ) and is making an angle β with a and c. Infact, set of all such points lie on a sector of a circle in the plane, Σ. We are interested in finding p such that it is equi-distant from a and c and lying towards one side of the line joining a and c (i.e. $\bar{ac}$).

(See Figure A.1)

Consider a plane(Σ) passing through a. Generic equation of Σ is given by-

$$(x - x_1)l' + (y - y_1)m' + (z - z_1)n' = 0 \tag{A.1}$$

where l', m' and n' are the direction ratios of the plane Σ.

Since b and c also lie in this plane, we can substitute them in the above equation to get the following equations.

$$(x_2 - x_1)l' + (y_2 - y_1)m' + (z_2 - z_1)n' = 0 \tag{A.2}$$

$$(x_3 - x_1)l' + (y_3 - y_1)m' + (z_3 - z_1)n' = 0 \tag{A.3}$$

Solving above equations, we get the following relationship.

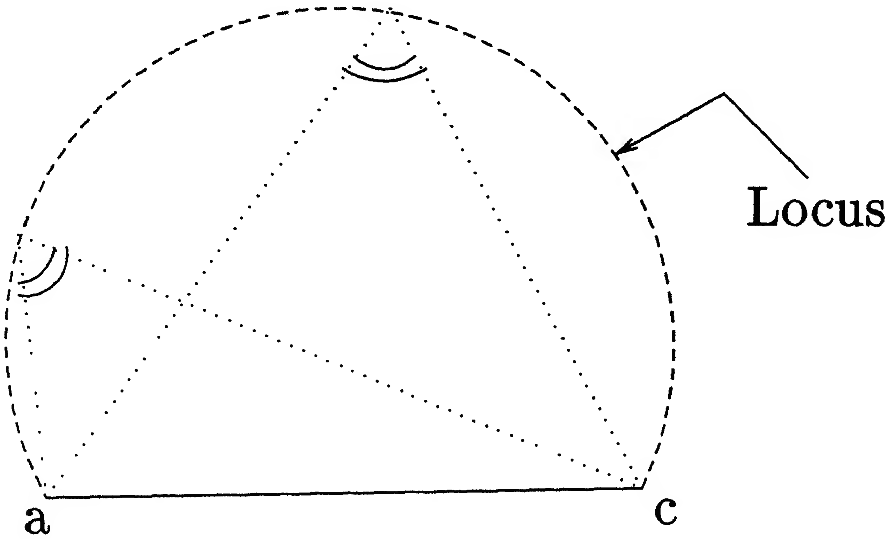

Figure A.1: Locus of the points making a fixed angle with two points

$$l' = K((y_2 - y_1)(z_3 - z_1) - (y_3 - y_1)(z_2 - z_1)) \tag{A.4}$$

$$m' = K((x_2 - x_1)(z_3 - z_1) - (x_3 - x_1)(z_2 - z_1)) \tag{A.5}$$

$$n' = K((x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)) \tag{A.6}$$

where K is any constant.

Now let us find the direction cosines of the bisector(psi) of $/barac$ such that it is lying in the plane N.

Let the d.c.s of the bisector be l'', m'' and n''

The bisector has to be perpendicular to ac and it must lie in N plane.

First condition the following equation.

$$(x_3 - x_1)l'' + (y_3 - y_1)m'' + (z_3 - z_1)n'' = 0 \tag{A.7}$$

while second condition gives the equation -

$$l' \cdot l'' + m' \cdot m'' + n' \cdot n'' = 0; \tag{A.8}$$

By reducing equations A.7 and A.8, we get

$$l'' = K'((y_3 - y_1) \cdot n' - (z_3 - z_1) \cdot m') \tag{A.9}$$

$$m'' = -K'((x_3 - x_1) \cdot n' - (z_3 - z_1) \cdot l') \tag{A.10}$$

$$n'' = K'((x_3 - x_1) \cdot m' - (y_3 - y_1) \cdot l') \tag{A.11}$$

where K' is a constant.

We can eliminate K' by using the fact that

$$l''^2 + m''^2 + n''^2 = 1 \tag{A.12}$$

We know that any point$(x_{new}, y_{new}, z_{new})$ a line which is passing through a point $p_{11}(x_{11}, y_{11}, z_{11})$ with d.c.s l_{new}, m_{new} and n_{new} is given by

$$x_{new} = d \cdot l_{new} + x_{11} \tag{A.13}$$

$$y_{new} = d \cdot m_{new} + y_{11} \tag{A.14}$$

$$z_{new} = d \cdot n_{new} + z_{11} \tag{A.15}$$

where d is the distance from p_{11} to p_{new}.

From the figure A.2, it is clear that we have to find a point which is at a distance of $\frac{\sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2 + (z_3 - z_1)^2)}}{2}$ from the midpoint of $\bar{ac}$. Infact, there are two such points. One of them lies on one side of the line $\bar{ac}$, while the other point lies on the opposite side of the first point.

Both the points that are lying on the bisector can be found by taking

$+\frac{\sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2 + (z_3 - z_1)^2)}}{2}$

and

$-\frac{\sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2 + (z_3 - z_1)^2)}}{2}$

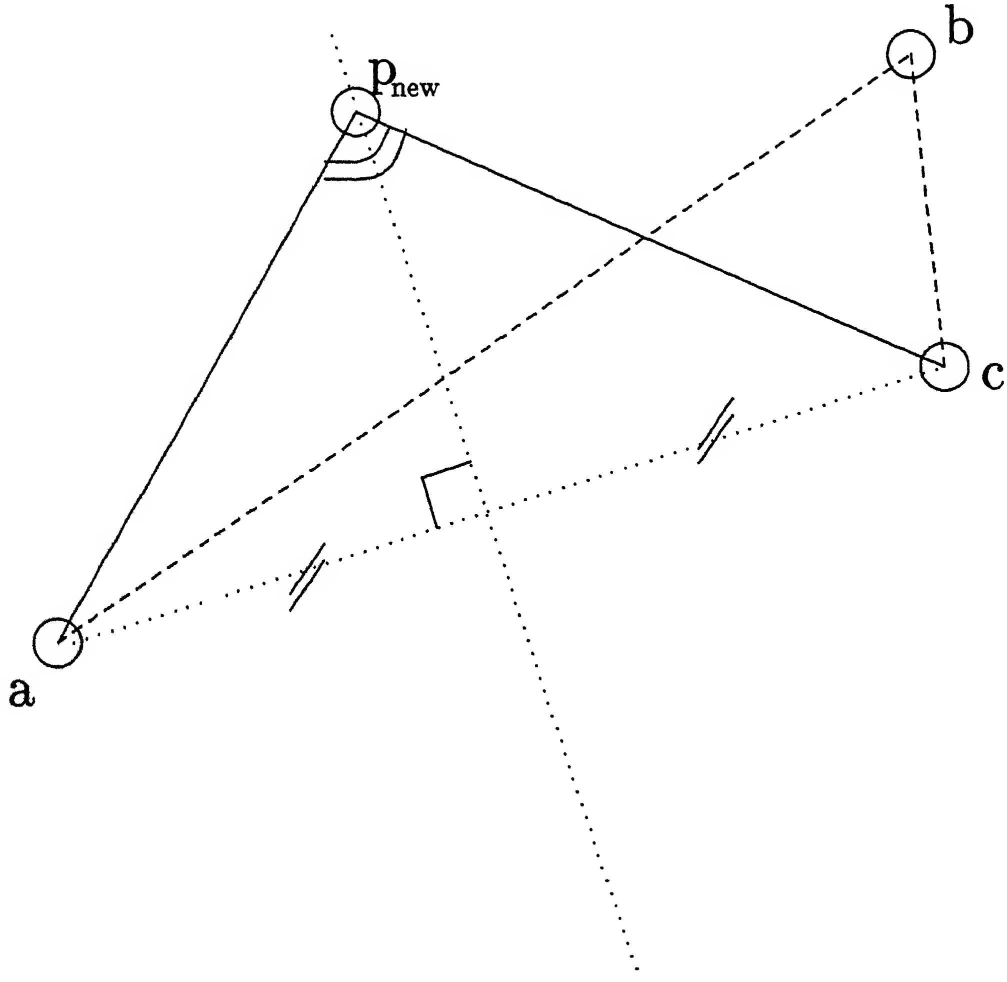

Figure A.2: Required Point on the Bisector

as the distances from the centre of $\bar{ac}$.

But we are interested in finding the point which is lying towards the side on which b is lying to the line $\bar{ac}$. This can be easily found as the required point is closer to b out of the two found points.